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**A NUMERICAL INVESTIGATION OF SCATTERING  
OF RADIATION OF AN ASYMMETRIC SOURCE  
BY CIRCULAR CYLINDER**

**N. Zitron and J. Davis**

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## ABSTRACT

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An asymptotic expansion obtained previously is employed to explore the limits of validity of plane wave and line source assumptions for the incident wave, in the problem of scattering of waves by an infinite cylinder. Detailed numerical calculations are made in the cases where the incident wave is a plane wave, line source radiation, and slotted cylinder radiation. Curves are presented showing the variation of the amplitude of the scattered wave with respect to spacing of the source and scatterer. The effect of the directivity of a source is discussed.

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# A NUMERICAL INVESTIGATION OF SCATTERING OF RADIATION OF AN ASYMMETRIC SOURCE BY CIRCULAR CYLINDER

N. Zitron and J. Davis

## Introduction

When radiation from a cylindrical source is incident on a cylindrical scatterer, the wave incident upon the scatterer and, consequently, the scattered wave will depend on the distance between the source and the scatterer. If, for example, the spacing is infinite, the incident wave will be a plane wave. If the spacing is large, but not infinite, the incident wave is equivalent to an isotropic cylindrical wave emanating from a line source or an equivalent isotropic source. As the source is brought closer to the scatterer, the directivity pattern of the source becomes more important and any anisotropies of the source must be taken into account.

The uncertainties in results based on a plane wave assumption are discussed in the field of microwaves by Kodis<sup>[1]</sup>, Wiles and Mc Lay<sup>[2]</sup>, Jordan and Mc Lay<sup>[3]</sup>, Subbaro and Mc Lay<sup>[4]</sup><sup>[5]</sup> and King and Wu<sup>[6]</sup>. The possibility of uncertainties in acoustics is discussed by Tamarkin<sup>[7]</sup> and by Lindsay<sup>[8]</sup>. Analytical and numerical work has been carried out by

Faran<sup>[9]</sup>, Froese and Wait<sup>[10]</sup>, and Shenderov<sup>[11]</sup> in the problem of line source radiation by a circular cylinder, in attempts to test the validity of the plane wave approximation. In this paper, the line source case is treated in greater detail and the case of anisotropy of the incident waves is also treated, with slotted cylinder radiation taken as an example. Consequently, some idea of the range of validity of the isotropy approximation for the incident wave is obtained.

The intuitive considerations discussed above have been made more precise by an expansion theorem of Karp and Zitron<sup>[12]</sup>. This theorem permits the representation of the field in a neighborhood disjoint from the circular cylinder circumscribed about the source region. The representation is in the form of an asymptotic series of inverse half-integral powers of  $kd$ , for large  $kd$ , where  $k$  is the propagation constant and  $d$  is the spacing. The coefficients in this series are linear combinations of products of derivatives of plane waves with respect to angle of incidence and derivatives of the complex scattering amplitude of the far field with respect to angle of incidence. Since the response of a scatterer to the derivative of a plane wave is the derivative of the response to a plane wave, this expansion theorem can be employed to obtain successive orders of approximation of the scattered field in terms of the spacing. The procedure is discussed elsewhere by Zitron and Karp<sup>[3]</sup> in greater detail. The principal purpose of this paper is to carry out numerical computations of these terms in order to obtain some idea of the ranges of validity of the plane wave

and line source approximations. The quantities to be computed are the amplitudes of the far fields scattered by conducting circular cylinders for various angles of observation in the cases where the incident radiation is a plane wave, line source or isotropic radiation, and slotted cylinder radiation as a function of the spacing between the source and the scatterer.

### An Analysis of the Problem

Consider a source or collection of sources of infinite length contained in a bounded cylindrical region. (See figure 1).

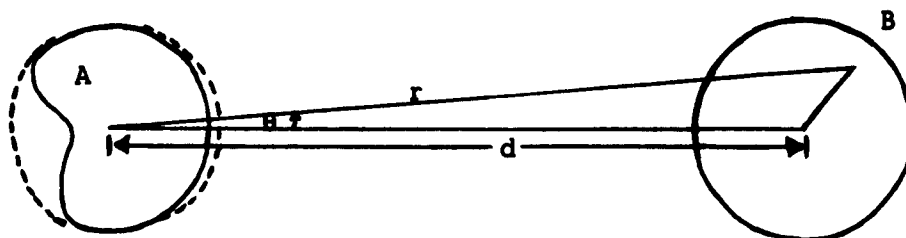


Figure 1

Consider a cylindrical neighborhood B at some distance "d" from A where "d" is the distance between the axis of the circular cylinder circumscribed about A and the axis of B. Let  $u(r, \theta)$  be a solution of the reduced wave equation

$$(1) \quad \nabla^2 u + k^2 u = 0$$

with time dependence  $e^{-i\omega t}$ . If  $f(\theta)$  is the complex far field amplitude of the sources in A, the field  $u(r, \theta)$  radiated by these sources has the following integral representation<sup>[14]</sup>.

$$(2) \quad u(r, \theta) = \int_{C_1} f(\beta) e^{ikr \cos(\theta - \beta)} d\beta$$

where  $C_1$  is the Sommerfeld contour for the Hankel function  $H_0^{(1)}(kr)$ . The following asymptotic expansion of  $u$  in the neighborhood  $B$  has been obtained by Karp and Zitron [12].

$$(3) \quad U(d, \theta) \simeq e^{ikd} \sum_{t=0}^m \frac{h(0)}{(2t)!} \frac{\Gamma(t + \frac{1}{2})}{(kd)^{t + \frac{1}{2}}}$$

$$(4) \quad \text{where} \quad h(0) \equiv \sum_{p=0}^{2t} \sum_{q=0}^{2t-p} \sum_{j=0}^q \frac{C_p^{2t} C_q^q}{q!} (\beta_s(0))^{(2t-p)} (\beta_s(0))^{(p)} f^{[q-j]}(0) v^{[j]}(0)$$

$$\text{and } C_j^q = \frac{q!}{j! (q-j)!},$$

$$v(\beta) = e^{ik(x \cos \beta + y \sin \beta)} \quad \beta_s = \frac{d\beta}{ds}$$

where  $x$  and  $y$  are defined in Figure 1, and where  $\beta$  is defined by

$$\cos \beta(s) = 1 + is^2 \quad \text{where } \beta(0) = 0, \text{ and } \beta_s(0) = +\sqrt{2} e^{\frac{-i\pi}{4}}.$$

The following differentiation symbols are used:

$$(p) \equiv \frac{d^p}{ds^p} \quad \text{and} \quad [j] \equiv \frac{d^j}{d\beta_j}.$$

The following explicit form of  $(\beta_s(0))^{(p)}$  obtained elsewhere [13] is included for convenience. For  $p$  even,  $(\beta_s(0))^{(p)} = (\beta_s(0))^{(2\nu)} = 2^{\nu + \frac{1}{2}} e^{-i(\nu + \frac{1}{2})\frac{\pi}{2}} \frac{(2\nu)!}{2^{2\nu} \nu!}$ .

For  $p$  odd,  $(\beta_s(0))^{(p)} = 0$ .

If  $u$  is represented asymptotically as

$$(5) \quad u_1 = \sum_{n=0}^m u_n \quad \text{where} \quad u_n = \frac{A_n}{(kd)^{n+\frac{1}{2}}},$$

the first few terms of (5) are

$$(6) \quad u_{10} = \frac{\sqrt{2\pi} e^{i(kd - \frac{\pi}{4})}}{(kd)^{\frac{1}{2}}} f(0) v(0)$$

$$(7) \quad u_{11} = \frac{\sqrt{2\pi} e^{i(kd - \frac{3\pi}{4})}}{2(kd)^{\frac{3}{2}}} \left[ \sum_{j=0}^2 f^{[2-j]}(0) v^{[j]}(0) + \frac{1}{4} f(0) v(0) \right]$$

$$(8) \quad u_{12} = \frac{\sqrt{2\pi} e^{i(kd - \frac{5\pi}{4})}}{32(kd)^{\frac{5}{2}}} \left[ 4 \sum_{i=0}^4 C_i^4 f^{[4-i]}(0) v^{[i]}(0) + 10 \sum_{j=0}^2 C_j^2 f^{[2-j]}(0) v^{[j]}(0) + \frac{9}{4} f(0) v(0) \right]$$

$$(9) \quad u_{13} = \frac{\sqrt{2\pi} e^{i(kd - \frac{7\pi}{4})}}{48(kd)^{\frac{7}{2}}} \left[ \sum C_j^6 f^{[6-j]}(0) v^{[j]}(0) + \frac{35}{4} \sum C_j^4 f^{[4-j]}(0) v^{[j]}(0) + \frac{259}{16} \sum C_j^2 f^{[2-j]}(0) v^{[j]}(0) + \frac{225}{64} f(0) v(0) \right]$$

where the subscript 1 denotes the incident field.

After obtaining the expansion of the radiated field in the neighborhood B, the next problem is to find the response of a circular cylinder to this incident field. The problem may be stated as follows:

Let  $u = u_1 + u_s$  where  $u_1$  is the incident field,  $u_s$  is the scattered field, and  $u$  is the total field. All fields satisfy the wave equation (1) and  $u$  satisfies the Sommerfeld radiation condition



$$(10) \quad \lim_{r \rightarrow \infty} \sqrt{r} \left\{ \frac{\partial u}{\partial r} - iku \right\} = 0 .$$

The boundary conditions considered here are the Dirichlet condition,  $u = 0$  on the cylinder boundary and the Neumann condition,  $\frac{\partial u}{\partial n} = 0$  on the boundary.

The Dirichlet condition corresponds to the electric field parallel to the cylinder axis in the electromagnetic case or a "soft" scatterer in the acoustic case. The Neumann condition corresponds to the magnetic field parallel to the cylinder axis in the electromagnetic case and a "hard" scatterer in the acoustic case.

The terms  $u_{in}$  of the incident field are in the form of linear combinations of derivatives of plane waves with respect to angle. The response to the derivative of a plane wave will be the derivative of the response to a plane wave and, therefore, the response to each term  $u_{in}$  of this incident field will be the same linear combination of derivatives of the response to a plane wave.

The response to a plane wave has the form

$$(11) \quad u_s = \sum_{n=-\infty}^{\infty} a_n H_n^{(1)}(kr) \cos n(\theta - \beta)$$

where the  $a_n$  are determined by the boundary conditions.

The far field may be represented in the form

$$(12) \quad u_s \sim H(kr) f_s(\theta, \beta)$$

$$\text{where } H(kr) = \sqrt{\frac{2}{\pi kr}} e^{i(kr - \frac{\pi}{4})}$$

and the complex scattering amplitude of the far field is given by

$$(13) \quad f_s(\theta, \beta) = \sum_{n=-\infty}^{\infty} a_n^{j-n} \cos n(\theta - \beta)$$

where  $\theta$  is the angle at which the scattered field is observed and  $\beta$  is the angle of incidence. If  $f_1(\beta)$  denotes the complex far field amplitude of the incident radiation, the scattered fields corresponding to the incident fields (6 - 9) may be written in the form

$$(14) \quad u_{s0} = H(kr) \frac{\sqrt{2\pi} e^{i(kd - \frac{\pi}{4})}}{(kd)^{\frac{1}{2}}} f_1(0) f_s(\theta, 0)$$

$$(15) \quad u_{s1} = H(kr) \frac{\sqrt{2\pi} e^{i(kd - \frac{3\pi}{4})}}{2(kd)^{\frac{1}{2}}} \left[ \sum_{j=0}^2 C_j^2 f_1^{[2-j]}(0) f_s^{[j]}(\theta, 0) + \frac{1}{4} f_2(0) f_s(\theta, 0) \right]$$

$$(16) \quad u_{s2} = H(kr) \frac{\sqrt{2\pi} e^{i(kd - \frac{5\pi}{4})}}{32(kd)^{\frac{1}{2}}} \left[ 4 \sum_{j=0}^4 C_j^4 f_1^{[4-j]}(0) f_s^{[j]}(\theta, 0) + 10 \sum_{j=0}^2 C_j^2 f_1^{[2-j]}(0) f_s^{[j]}(\theta, 0) + \frac{9}{4} f_1(0) f_s(\theta, 0) \right]$$

$$(17) \quad u_{s3} = H(kr) \frac{\sqrt{2\pi} e^{i(kd - \frac{7\pi}{4})}}{48(kd)^{\frac{1}{2}}} \left[ 1 \cdot \sum_{j=0}^6 C_j^6 f_1^{[6-j]}(0) f_s^{[j]}(\theta, 0) + \frac{35}{4} \sum_{j=0}^4 C_j^4 f_1^{[4-j]}(0) f_s^{[j]}(\theta, 0) + \frac{259}{16} \sum_{j=0}^2 C_j^2 f_1^{[2-j]}(0) f_s^{[j]}(\theta, 0) + \frac{225}{64} f_1(0) f_s(\theta, 0) \right] .$$

A measure of caution should be observed in attempting to apply these results .  
The first correction term  $u_{s1}$  given by (15) is of order  $(kd)^{-\frac{3}{2}}$ , but the  
rescattering of the term  $u_{s0}$  by the source contributes a term of order  $(kd)^{-1}$  .<sup>\*</sup>  
In an experimental situation, this term, which depends upon the structure of  
the source, would have to be considered unless the rescattering effect of the  
source is small or the source is shielded against radiation from the scatterer.

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<sup>\*</sup> See reference [13] for details.

## Method of Computation

In the numerical calculation it was convenient to use the relation

$$(18) \quad u_s(\theta, r) = \left(\frac{1}{kr}\right)^{\frac{1}{2}} e^{i(kr - \frac{\pi}{4})} F(\theta)$$

to normalize the far field amplitude  $F(\theta)$ . A general program was written for the Control Data Corporation 1604, in which the modulus of  $F(\theta)$  was calculated for various values of  $ka$ ,  $kd$ ,  $\theta$  and  $f_1^{(i)}(0)$ , using formulas (14-17). The incident wave was normalized in such a way that a standard field was produced at the position of the center of the scatterer in the absence of the scatterer.

From formulas (6-9) above, the following expansion was obtained for  $u_{10} \Big|_{\substack{x=0 \\ y=0}}$ , the field at this position:

$$(19) \quad u_{10} \Big|_{\substack{x=0 \\ y=0}} \sim \frac{\sqrt{2\pi} e^{i(kd - \frac{\pi}{4})}}{\sqrt{kd}} f_1(0) \left\{ 1 - \frac{1}{2kd} \left[ \frac{1}{4} + \frac{f_1^{[2]}(0)}{f_1(0)} \right] \right. \\ \left. - \frac{1}{8(kd)^2} \left[ \frac{f_1^{[4]}(0)}{f_1(0)} + \frac{5}{2} \frac{f_1^{[2]}(0)}{f_1(0)} + \frac{9}{16} \right] \right. \\ \left. + \frac{1}{48(kd)^3} \left[ \frac{f_1^{[6]}(0)}{f_1(0)} + \frac{35}{4} \frac{f_1^{[4]}(0)}{f_1(0)} + \frac{259}{16} \frac{f_1^{[2]}(0)}{f_1(0)} + \frac{225}{16} \right] \right\} \\ + \dots$$

$f_1(\beta)$  is normalized so that  $u_{10} \Big|_{\substack{z=0 \\ y=0}} = 1$ ; in other words the ratio  $\frac{f_1(\beta)}{u_{10} \Big|_{\substack{x=0 \\ y=0}}}$

was used instead of  $f_1(\beta)$ . The introduction of this normalization and consequent

simplification gave the following formula for  $F(\theta)$  :

$$(20) \quad F(\theta) = \frac{N}{D}, \text{ where}$$

$$\begin{aligned} N \sim \sqrt{\frac{2}{\pi}} \left\{ f_s(\theta, 0) - \frac{1}{2kd} \left[ \sum_{j=0}^2 C_j^2 \frac{f_1^{[2-j]}(0)}{f_1(0)} f_s^{[j]}(\theta, 0) + \frac{1}{4} f_s(\theta, 0) \right] \right. \\ - \frac{1}{8(kd)^2} \left[ \sum_{j=0}^4 C_j^4 \frac{f_1^{[4-j]}(0)}{f_1(0)} f_s^{[j]}(\theta, 0) + \frac{8}{2} \sum_{j=0}^2 C_j^2 \frac{f_1^{[2-j]}(0)}{f_1(0)} f_s^{[j]}(\theta, 0) + \frac{9}{16} f_s(\theta, 0) \right] \\ + \frac{1}{48(kd)^3} \left[ \sum_{j=0}^6 C_j^6 \frac{f_1^{[6-j]}(0)}{f_1(0)} f_s^{[j]}(\theta, 0) + \frac{35}{4} \sum_{j=0}^4 C_j^4 \frac{f_1^{[4-j]}(0)}{f_1(0)} f_s^{[j]}(\theta, 0) \right. \\ \left. \left. + \frac{259}{16} \sum_{j=0}^2 C_j^2 \frac{f_1^{[2-j]}(0)}{f_1(0)} f_s^{[j]}(\theta, 0) + \frac{225}{64} f_s(\theta, 0) \right] \right\}, \text{ and} \end{aligned}$$

$$D = \left( \frac{\sqrt{kd}}{\sqrt{2\pi} e^{i(kd - \frac{\pi}{4})} f_1(0)} \right) u_{10} \Big|_{\substack{x=0 \\ y=0}}.$$

It was assumed in all of these calculations that the scatterer was a circular cylinder of radius  $a$ , satisfying either Dirichlet or Neumann boundary conditions on its surface. In these cases, the coefficients  $a_n$  in formula (11) are easily derived:

$$a_n = -i^m \frac{J_m(ka)}{H_m^{(1)}(ka)} \text{ for the Dirichlet condition, or}$$

$$a_n = -i^m \frac{J'_m(ka)}{H_m^{(1)'}(ka)} \text{ for the Neumann condition,}$$

where  $J_m$  and  $H_m^{(1)}$  are Bessel and Hankel functions respectively. Tables [15] are available of Bessel and Hankel functions expressed in terms of a magnitude  $C_n$  and a phase  $\delta_n$ . The relevant properties are these:

$$J_n(ka) = C_n(ka) \sin \delta_n(ka)$$

$$H_n^{(1)}(ka) = -i C_n(ka) e^{i\delta_n(ka)} .$$

These relations and some further simplifications give the following formula for the Dirichlet boundary condition:

$$(21) \quad f_s(\theta, \beta) = i \sum_{n=0}^{\infty} \epsilon_n \sin \delta_n e^{-i\delta_n} \cos n(\theta - \beta), \text{ where}$$

$$\epsilon_n = \min(2, n+1) .$$

Similarly, for the Neumann condition, there exists a set of  $\delta'_n(ka)$ 's, which give:

$$f_s(\theta, \beta) = i \sum_{n=0}^{\infty} \epsilon_n \sin \delta'_n e^{-i\delta'_n} \cos n(\theta - \beta) .$$

In calculating  $N$  and  $D$ , at least two terms, and not more than four terms of their respective expansions, were used. Terms were calculated in ascending powers of  $\frac{1}{kd}$ , stopping after a term, if this term was smaller in magnitude than

a certain percentage (usually between 1% and 10%) of the total sum. For  $N$  at each angle and for  $D$ , a record is printed indicating whether or not this convergence criterion had been met, and if it had been met, how many terms were needed.

In the above,  $N$  and  $D$  were calculated separately, and then  $N$  was divided by  $D$ . An alternative method, tried for sources for which  $f(\beta)$  is even, consisted of dividing  $N$  by  $D$  analytically, retaining terms up to order  $\frac{1}{(kd)^3}$ . For  $f(\beta)$  even, the following holds:

$$\begin{aligned}
 (22) \quad F(\theta) \sim & \sqrt{\frac{2}{\pi}} \left\{ f_s(\theta, 0) - \frac{1}{2(kd)} f_s(\theta, 0) \right. \\
 & - \frac{1}{8(kd)^2} \left[ f_s^{[4]}(\theta, 0) + 2f_s^{[2]}(\theta, 0) + 4f_s(\theta, 0) \frac{f_1^{[2]}(0)}{f_1(0)} \right] \\
 & + \frac{1}{48(kd)^3} \left[ f_s^{[6]}(\theta, 0) + 8f_s^{[4]}(\theta, 0) + 13f_s^{[2]}(\theta, 0) \frac{f_1^{[4]}(0)}{f_1(0)} + 12f_s(\theta, 0) \frac{f_1^{[4]}(0)}{f_1(0)} \right. \\
 & \left. \left. + 27f_s(\theta, 0) \frac{f_1^{[2]}(0)}{f_1(0)} - 12 \left( \frac{f_1^{[2]}(0)}{f_1(0)} \right)^2 \right] \right\}.
 \end{aligned}$$

The results calculated in this manner agreed well with the results of the previous method; moreover, in the case of derivatives  $f_1^{(2n)}(0)$  of large magnitude the last method converges faster.

Three different types of sources were considered: a plane wave, a line source ( $f(\beta) = 1$ ), and a slotted cylinder. This latter source was reported on

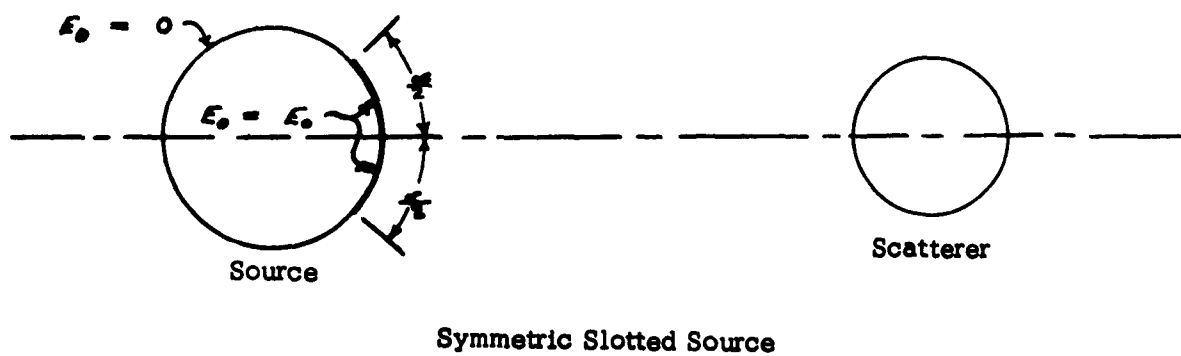
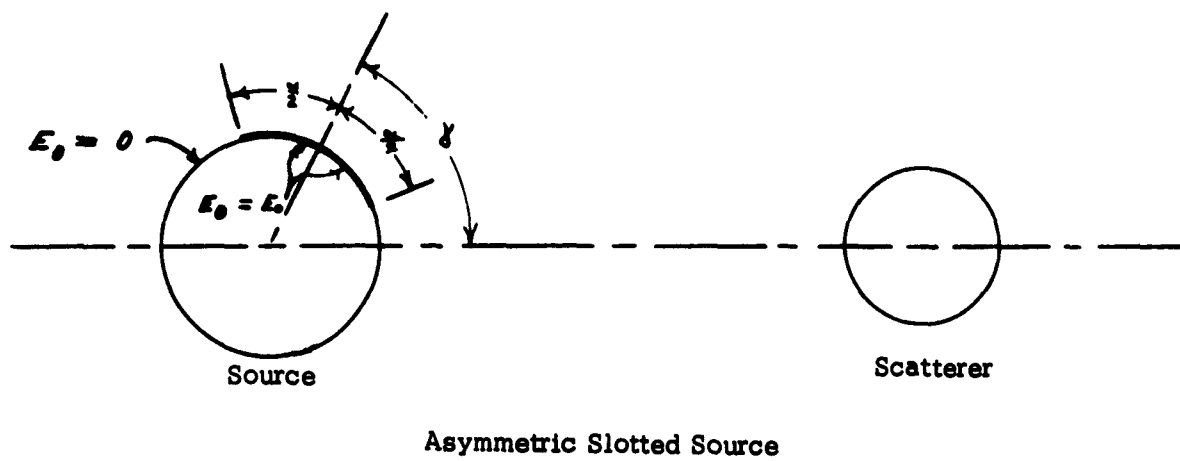


Figure 2



by Papas and King<sup>[16]</sup>, and consists of a circular cylinder of radius  $c$ , with a tangential electric field  $E_\theta(\theta)$  prescribed on its surface, where

$$E_\theta(\theta) = E_0 \quad \text{for } |\theta - \gamma| < \frac{\alpha}{2} \quad (\text{the "slot"})$$

$$\text{and} \quad E_\theta(\theta) = 0 \quad \text{for } |\theta - \gamma| \geq \frac{\alpha}{2} .$$

The letter  $\gamma$  stands for the orientation of the slot, and  $\alpha$  stands for the angular width of the slot (see figure 2). In this problem, the  $z$ -component of the magnetic field satisfies the scalar wave equation with Neumann boundary conditions. Values of  $f_1^{[n]}(0)$  were computed from the formula for the far field given by Papas and King<sup>[16]</sup>. Values of  $0^\circ$  and  $90^\circ$  were used for  $\gamma$ , while  $\alpha$  was set at .001 radius.

Checks were made at  $ka = 1.$  and  $3.4$  for the line source, using vector summation rules developed by Lowan, Morse, Feshbach, and Lax<sup>[15]</sup> and Faran<sup>[9]</sup>. This approach results in a convergent series for  $F(\theta)$ , in ascending powers of  $ka$ .

#### A Discussion of the Results

The complex scattering amplitude of the far field was calculated for all integral values of  $kd$  from 1 to 10 and also  $ka = 3.4$ , where " $a$ " is the radius of the scatterer, and for values  $\frac{d}{a} = 2, 3, 5, 10, 30, 50, 100$ , and  $\infty$  for the line source and the slotted cylinder. The case  $\frac{d}{a} = \infty$  represents plane wave incidence. Both the Dirichlet and the Neumann conditions were applied in

the line source calculations. Curves and tables for  $ka = 1, 3.4$ , and  $10$  are included.

The first case considered is that of the line source (Tables 1 - 4 and Figures 3 - 7) and its limiting case, the plane wave. In this case, the rate of convergence of the expansion in half-integral powers of  $kd$  is good in the sense that for the Neumann condition only four terms of the asymptotic expansion were needed to maintain a deviation of less than 2% from the more exact formula of Lowan, Morse, Feshbach, and Lax<sup>[15]</sup> for  $d > 2a$  in the cases  $ka = 1$  and  $ka = 3.4$  and for  $d > 8a$  in the case  $ka = 10$ . Table 5 shows the minimum values of  $\frac{d}{a}$  for which one, two, or three terms are sufficient to meet specified cutoff criteria. The cutoff criteria employed are termination of calculation of the series when the addition of the last term calculated changed the result by less than 1%. The curve obtained here for  $kd = 6.8$  differs from that of Faran<sup>[9]</sup>. A computational check of the analytical expression from which Faran's curves were obtained seems to indicate a numerical error in his curve.

The next case considered was the slotted cylinder source (see figures 8, 9) with plane symmetry, such that  $kc = 8$  and with an aperture of .001 radians. An examination of formulas (20) and (22) reveals the effect of the shape or directivity terms, i. e. the terms that depend upon derivatives of  $f_1(\beta)$ . The directivity effect is of second order in  $\frac{1}{kd}$ . The first order term in the far field is cancelled by another first order term in the normalization. Thus, a symmetrically oriented source may be approximated by a line source for large  $kd$ . The approximation for this slotted source is good to 1% for  $\frac{d}{a} > 8$ .

The rate of convergence of the series was much slower for the slotted source than for the line source at the same distance, since the derivatives of  $f_1(\beta)$  (Table 6) are somewhat large. For small  $kd$ , moreover, the series failed to converge at all in some cases. Thus, for such small  $kd$ , the addition of further terms in the series would not improve the accuracy for the slotted cylinder as it would for the line source. Furthermore, since the difference between the scattered fields of the line source and the slotted cylinder is of second order in  $\frac{1}{kd}$ , values of  $kd$  small enough to produce a large directivity effect were also small enough to cause convergence difficulties. A set of minimum values of  $ka$  for specified accuracy is contained in Table 5.

A more extreme directivity effect was obtained by rotating the source  $90^\circ$ . The amplitude of the derivatives of  $f_1(\beta)$  was thus increased and the rate of convergence decreased. Another directivity effect (Figures 10, 11) was a rotation of the scattering pattern by an angle of  $\frac{c}{d}$  radians for small  $c/d$ . The magnitude of the directivity effect for the rotated slotted cylinder can be appreciable for large  $d/a$ . For example, if  $ka = 1$  and  $\frac{d}{a} = 100$ , the difference between the scattered field of a line source and a rotated slotted cylinder can be as high as 8% at some angles and as high as 25% for  $ka = 3.4$  and  $\frac{d}{a} = 20$ .

These results shed some light upon the reasons for the relative success of the aforementioned experimenters<sup>[1-5], [7]</sup> in obtaining agreement between their results and the theory. Their sources were sufficiently far away for the values

of wave length and cylinder radii employed. The symmetric nature of the source reduced the directivity effects considerably and made a line source approximation reasonable.

#### Concluding Remarks

In conclusion, the results may be summarized as follows:

1. In the case of a symmetrical source oriented symmetrically with respect to the scatterer, the line source approximation is often good for relatively small spacing between source and scatterer.
2. In the asymmetric case, the directivity is a much more important factor than in the symmetric case.
3. The success of various experimenters in obtaining agreement with experiment has been due in part to their use of symmetric sources and also to the relatively large spacing between sources and scatterers.
4. The asymptotic expansion upon which these calculations are based exhibits convergence difficulties if  $d$  is sufficiently small.

TABLE I

THE FARFIELD AMPLITUDE  $F(\theta)$  $ka = 1.0$ 

θ	DIRICHLET CONDITION					NEUMANN CONDITION					SLOTTED SOURCE $K_c = 8, d = .001$			
	PLANE WAVE	LINE SOURCE				PLANE WAVE	LINE SOURCE				SYMMETRIC		ROTATED	
		kd=20.	kd=10.	kd=5.	kd=2.		kd=20.	kd=10.	kd=5.	kd=2.	δ = 0	δ = 0	δ = 90	kd=100
0°	1.375	1.356	1.337	1.299	1.20	0.512	0.530	0.548	0.587	0.711	0.539	0.60	0.511	0.511
10°	1.360	1.342	1.323	1.288	1.19	0.499	0.516	0.534	0.572	0.693	0.525	0.58	0.512	0.48
20°	1.319	1.302	1.285	1.253	1.17	0.462	0.478	0.495	0.530	0.641	0.486	0.54	0.487	0.44
30°	1.254	1.240	1.226	1.200	1.137	0.408	0.422	0.437	0.467	0.565	0.430	0.48	0.442	0.38
40°	1.172	1.162	1.152	1.134	1.094	0.353	0.364	0.375	0.397	0.464	0.369	0.40	0.386	0.33
50°	1.083	1.077	1.072	1.063	1.049	0.318	0.323	0.328	0.339	0.369	0.325	0.343	0.338	0.31
60°	0.994	0.993	0.992	0.993	1.005	0.322	0.322	0.321	0.319	0.303	0.322	0.321	0.321	0.33
70°	0.915	0.918	0.922	0.931	0.966	0.367	0.363	0.358	0.347	0.298	0.361	0.346	0.345	0.39
80°	0.851	0.859	0.866	0.882	0.935	0.435	0.429	0.423	0.409	0.352	0.426	0.41	0.403	0.47
90°	0.806	0.816	0.827	0.848	0.914	0.507	0.501	0.495	0.481	0.432	0.498	0.48	0.473	0.54
100°	0.778	0.791	0.803	0.828	0.901	0.573	0.567	0.562	0.550	0.512	0.565	0.55	0.542	0.60
110°	0.766	0.779	0.792	0.818	0.895	0.628	0.623	0.618	0.608	0.583	0.621	0.60	0.603	0.65
120°	0.763	0.776	0.790	0.816	0.897	0.669	0.665	0.661	0.653	0.637	0.663	0.65	0.651	0.68
130°	0.766	0.779	0.792	0.819	0.896	0.699	0.695	0.691	0.685	0.679	0.693	0.68	0.686	0.71
140°	0.771	0.784	0.796	0.823	0.900	0.718	0.714	0.711	0.706	0.708	0.714	0.701	0.710	0.72
150°	0.777	0.788	0.801	0.827	0.904	0.729	0.725	0.723	0.719	0.725	0.725	0.714	0.724	0.73
160°	0.781	0.792	0.805	0.830	0.907	0.735	0.731	0.729	0.725	0.73	0.731	0.722	0.732	0.735
170°	0.784	0.795	0.807	0.833	0.908	0.737	0.734	0.731	0.728	0.74	0.734	0.725	0.736	0.737
180°	0.784	0.796	0.808	0.834	0.909	0.738	0.734	0.732	0.729	0.74	0.734	0.726	0.737	0.737

TABLE II

THE FARFIELD AMPLITUDE  $F(\theta)$  $ka = 3.4$ 

$\theta$	DIRICHLET CONDITION				NEUMANN CONDITION				SLOTTED SOURCE $kc = \frac{8}{d} = \frac{8}{1.001}$		
	PLANE WAVE	LINE SOURCE			PLANE WAVE	LINE SOURCE			SYMMETRIC $\delta = 0^\circ$	ROTATED $\delta = 90^\circ$	$kd=68$
		$kd=68$	$kd=17$	$kd=13.6$	$kd=6.8$	$kd=68$	$kd=17$	$kd=13.6$	$kd=6.8$	$kd=17$	$F(\theta)$
$0^\circ$	3.462	3.387	3.145	3.06	2.61	2.228	2.309	2.532	2.596	2.75	2.18
$10^\circ$	3.178	3.118	2.925	2.86	2.50	2.072	2.136	2.312	2.364	2.48	2.32
$20^\circ$	2.449	2.430	2.371	2.35	2.22	1.739	1.750	1.783	1.794	1.81	2.11
$30^\circ$	1.607	1.645	1.752	1.78	1.90	1.495	1.441	1.273	1.220	1.11	1.73
$40^\circ$	1.122	1.190	1.380	1.44	1.67	1.369	1.284	1.005	0.907	0.75	1.43
$50^\circ$	1.149	1.185	1.305	1.35	1.56	1.127	1.049	0.801	0.715	0.59	1.24
$60^\circ$	1.250	1.259	1.305	1.33	1.49	0.691	0.644	0.498	0.448	0.37	0.93
$70^\circ$	1.242	1.245	1.273	1.29	1.42	0.459	0.468	0.484	0.483	0.50	0.49
$80^\circ$	1.199	1.208	1.245	1.261	1.38	0.835	0.845	0.858	0.855	0.88	0.48
$90^\circ$	1.198	1.210	1.252	1.268	1.38	1.170	1.164	1.135	1.123	1.13	0.91
$100^\circ$	1.229	1.241	1.279	1.294	1.40	1.272	1.257	1.207	1.190	1.17	1.18
$110^\circ$	1.259	1.269	1.306	1.320	1.42	1.192	1.178	1.142	1.132	1.11	1.22
$120^\circ$	1.276	1.287	1.327	1.342	1.44	1.074	1.074	1.085	1.090	1.10	1.11
$130^\circ$	1.288	1.301	1.346	1.362	1.46	1.053	1.070	1.126	1.145	1.18	1.02
$140^\circ$	1.300	1.315	1.364	1.382	1.49	1.132	1.155	1.225	1.247	1.29	1.06
$150^\circ$	1.312	1.328	1.381	1.400	1.51	1.229	1.249	1.307	1.326	1.36	1.16
$160^\circ$	1.322	1.338	1.393	1.413	1.53	1.293	1.306	1.346	1.360	1.382	1.26
$170^\circ$	1.328	1.345	1.401	1.421	1.54	1.322	1.329	1.355	1.365	1.379	1.31
$180^\circ$	1.330	1.347	1.403	1.423	1.54	1.329	1.333	1.355	1.365	1.375	1.33

TABLE III

## THE FAR FIELD AMPLITUDE

 $F(\theta)$ ,  $ka = 10$ .

$\theta$	NEUMANN CONDITION										SYMMETRIC SLOTTED SOURCE			
	PLANE WAVE		LINE SOURCE										$\delta = 0^\circ$	
			$kd = 500.$		$kd = 200.$		$kd = 100.$		$kd = 500.$		$kd = 200.$		$kd = 100.$	
	$F(\theta)$	$F(\theta+90^\circ)$	$F(\theta)$	$F(\theta+90^\circ)$	$F(\theta)$	$F(\theta+90^\circ)$	$F(\theta)$	$F(\theta+90^\circ)$	$F(\theta)$	$F(\theta+90^\circ)$	$F(\theta)$	$F(\theta+90^\circ)$	$F(\theta)$	$F(\theta+90^\circ)$
$0^\circ$	7.259	2.012	7.374	2.002	7.534	1.985	7.74	1.956	7.376	2.001	7.539	1.984	7.78	1.951
$5^\circ$	6.442	1.855	6.509	1.843	6.602	1.826	6.742	1.803	6.510	1.843	6.609	1.826	6.74	1.800
$10^\circ$	4.498	1.761	4.446	1.767	4.366	1.779	4.246	1.798	4.445	1.767	4.361	1.780	4.22	1.803
$15^\circ$	2.443	1.951	2.281	1.963	2.037	1.980	1.65	2.001	2.278	1.963	2.020	1.981	1.57	2.006
$20^\circ$	0.889	2.103	0.753	2.103	0.554	2.103	0.33	2.099	0.751	2.103	0.540	2.103	0.34	2.098
$25^\circ$	1.017	2.058	1.036	2.052	1.078	2.046	1.175	2.037	1.037	2.052	1.081	2.046	1.19	2.036
$30^\circ$	1.826	1.982	1.782	1.986	1.717	1.994	1.61	2.014	1.781	1.986	1.712	1.995	1.59	2.015
$35^\circ$	1.680	2.040	1.611	2.054	1.509	2.074	1.33	2.109	1.610	2.054	1.501	2.075	1.30	2.112
$40^\circ$	0.591	2.152	0.553	2.164	0.498	2.178	0.41	2.200	0.552	2.163	0.494	2.179	0.40	2.203
$45^\circ$	1.058	2.187	1.063	2.191	1.067	2.197	1.06	2.206	1.063	2.191	1.067	2.197	1.06	2.208
$50^\circ$	1.825	2.147	1.793	2.151	1.745	2.159	1.659	2.173	1.793	2.151	1.741	2.159	1.64	2.175
$55^\circ$	1.556	2.122	1.515	2.132	1.456	2.148	1.359	2.179	1.515	2.132	1.452	2.149	1.34	2.183
$60^\circ$	0.997	2.154	0.996	2.168	0.995	2.189	0.992	2.226	0.996	2.168	0.995	2.191	0.99	2.232
$65^\circ$	1.541	2.209	1.545	2.221	1.548	2.240	1.544	2.268	1.545	2.221	1.548	2.240	1.54	2.273
$70^\circ$	1.919	2.240	1.899	2.248	1.866	2.261	1.810	2.280	1.898	2.248	1.864	2.261	1.80	2.283
$75^\circ$	1.648	2.238	1.625	2.245	1.592	2.255	1.543	2.272	1.625	2.245	1.591	2.255	1.53	2.276
$80^\circ$	1.453	2.222	1.459	2.232	1.469	2.245	1.483	2.272	1.459	2.232	1.471	2.247	1.49	2.276
$85^\circ$	1.811	2.211	1.818	2.224	1.828	2.244	1.835	2.279	1.819	2.224	1.828	2.246	1.84	2.285
$90^\circ$	2.012	2.208	2.002	2.223	1.985	2.245	1.956	2.284	2.001	2.223	1.984	2.247	1.951	2.290

TABLE IV

THE FARFIELD AMPLITUDE  $F(\theta)$  $ka = 10.$ 

	DIRICHLET CONDITION										NEUMANN		ROTATED		SLOTTED	
	PLANE WAVE		LINE SOURCE								Line Source		SOURCE			
			$kd = 500.$		$kd = 200.$		$kd = 100.$				$kd = 200.$		$kc = 8.$		$kd = 200, \delta = 90^\circ$	
	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$	$F(\theta)$	$F(\theta + 90^\circ)$
0°	8.955	1.940	8.838	1.943	8.642	1.947	8.28	1.954	7.534	1.985	7.26	1.91	7.26	1.91	7.26	1.97
5°	7.644	1.973	7.568	1.976	7.440	1.980	7.20	1.987	6.602	1.826	7.31	1.88	7.31	1.88	5.52	1.82
10°	4.485	1.997	4.528	2.000	4.588	2.007	4.690	2.020	4.366	1.779	5.63	1.71	5.63	1.71	3.08	1.915
15°	1.676	2.024	1.882	2.028	2.177	2.036	2.632	2.048	2.037	1.980	3.20	1.82	3.20	1.82	1.08	2.103
20°	2.084	2.053	2.140	2.058	2.248	2.066	2.470	2.083	0.554	2.103	1.17	2.03	1.17	2.03	0.57	2.127
25°	2.284	2.079	2.285	2.084	2.301	2.094	2.363	2.111	1.078	2.046	0.71	2.05	0.71	2.05	1.42	2.050
30°	1.702	2.101	1.730	2.108	1.777	2.118	1.867	2.137	1.717	1.994	1.51	1.96	1.51	1.96	1.71	2.062
35°	1.652	2.123	1.678	2.130	1.717	2.142	1.785	2.162	1.509	2.074	1.74	1.98	1.74	1.98	1.07	2.166
40°	1.858	2.144	1.857	2.152	1.859	2.164	1.871	2.186	0.498	2.178	1.02	2.10	1.02	2.10	0.56	2.227
45°	1.743	2.162	1.744	2.171	1.750	2.184	1.762	2.207	1.067	2.197	0.46	2.170	0.46	2.170	1.52	2.208
50°	1.670	2.179	1.679	2.188	1.692	2.202	1.714	2.226	1.745	2.159	1.50	2.151	1.50	2.151	1.72	2.175
55°	1.771	2.194	1.773	2.203	1.776	2.218	1.780	2.244	1.456	2.148	1.67	2.120	1.67	2.120	1.19	2.191
60°	1.797	2.207	1.794	2.217	1.792	2.232	1.791	2.259	0.995	2.189	1.06	2.142	1.06	2.142	1.26	2.240
65°	1.772	2.217	1.774	2.228	1.778	2.244	1.782	2.272	1.548	2.240	1.15	2.199	1.15	2.199	1.82	2.268
70°	1.812	2.226	1.814	2.237	1.818	2.253	1.822	2.282	1.866	2.261	1.75	2.240	1.75	2.240	1.81	2.271
75°	1.863	2.233	1.862	2.244	1.863	2.261	1.863	2.290	1.592	2.255	1.73	2.248	1.73	2.248	1.52	2.258
80°	1.880	2.238	1.880	2.249	1.882	2.266	1.884	2.296	1.469	2.245	1.39	2.240	1.39	2.240	1.68	2.252
85°	1.901	2.241	1.904	2.253	1.908	2.270	1.914	2.300	1.828	2.244	1.58	2.238	1.58	2.238	1.99	2.251
90°	1.940	2.242	1.943	2.254	1.947	2.271	1.954	2.301	1.985	2.245	1.91	2.243	1.91	2.243	1.97	2.243



TABLE V

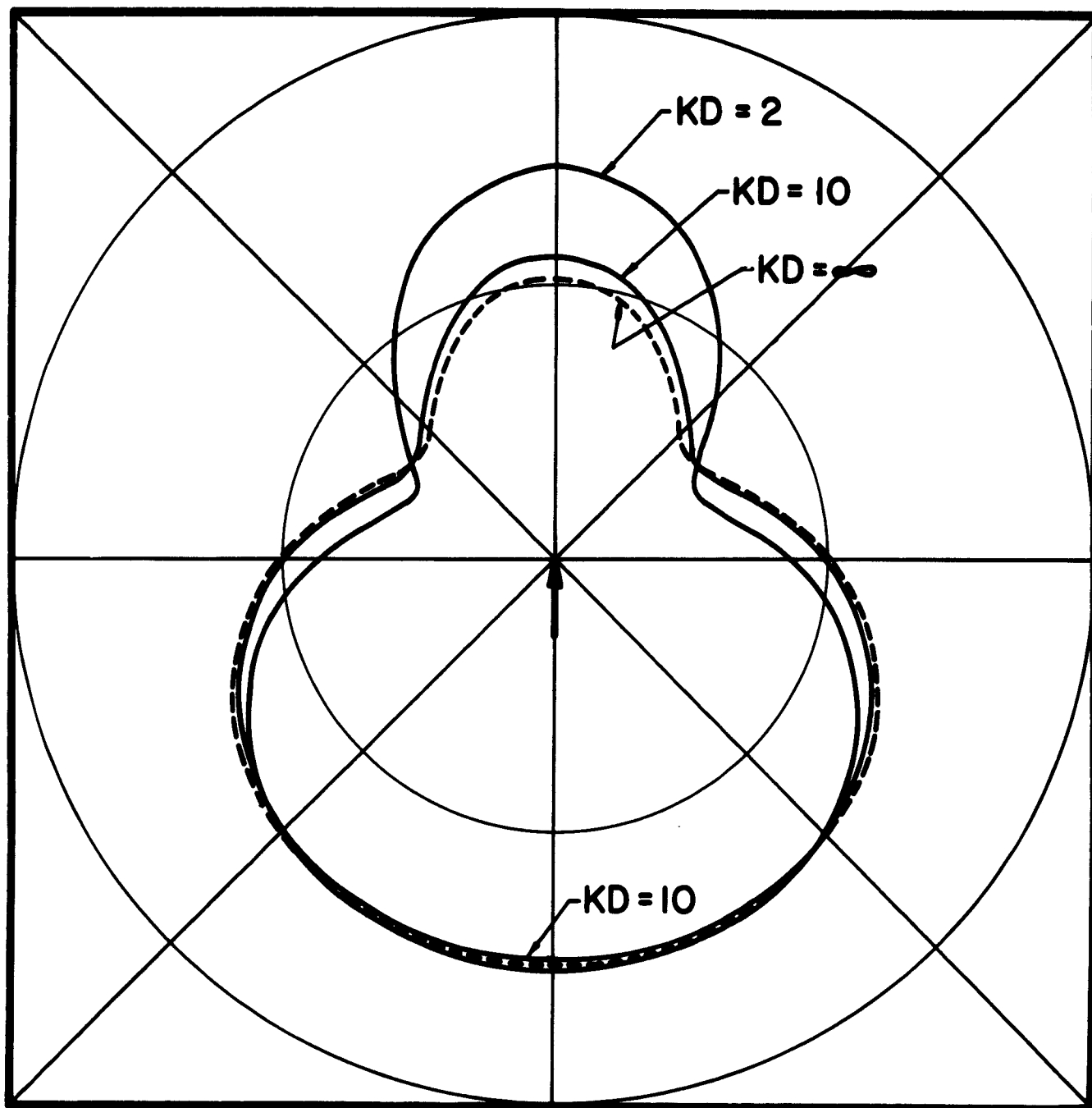
An entry in this table gives approximately the lowest value of  $k_d$  for which the indicated number of terms provides an error less than  $P$

Terms Used	P	ka = 1.0			ka = 3.4			ka = 10.0		
		Dirichlet Condition*	Neumann Condition*	Symmetric Slotted Source	Dirichlet Condition*	Neumann Condition*	Symmetric Slotted Source	Dirichlet Condition*	Neumann Condition*	Symmetric Slotted Source
Plane Wave	10%	3	7	15	15	15	15	50	70	70
	1%	30	70	70	100	70	70	500	700	700
Plane Wave Plus One Term	10%	2	4	10	4	4	10	10	20	30
	1%	3	7	40	15	15	40	30	40	50
Plane Wave Plus Two Terms	10%	1	2	--	2	3	8	5	10	10
	1%	$\frac{3}{2}$	3	--	5	8	15	10	20	20

TABLE VI  
SLOTTED SOURCE DERIVATIVES  $f_1^{[n]}$

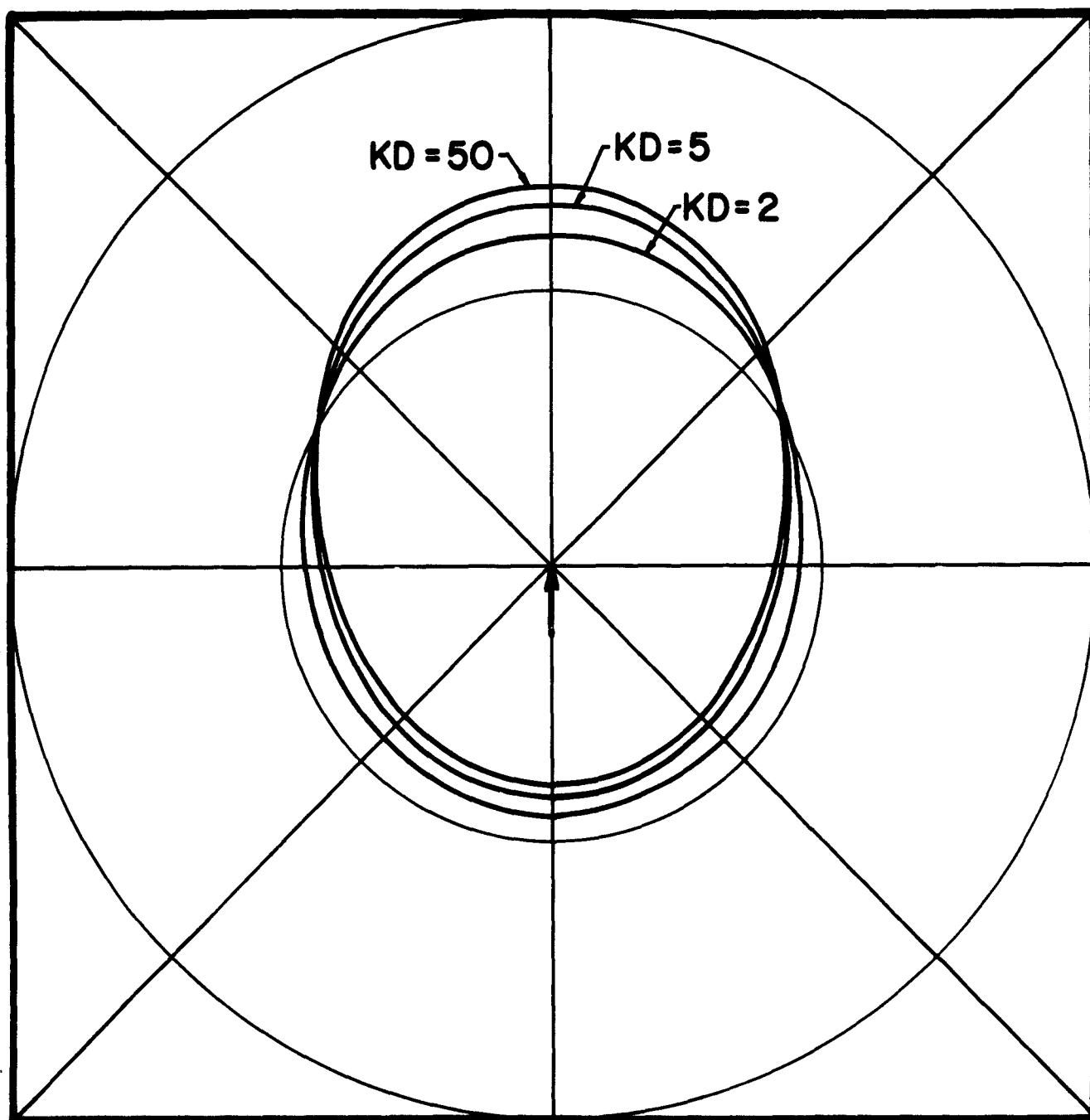
For  $kc = 8.0$ ,  $\alpha = .001$

n	$\gamma = 0^\circ$		$\gamma = 90^\circ$	
	$R_e(f_1^{[n]}(0))$	$I_{mj}(f_1^{[n]}(0))$	$R_e(f_1^{[n]}(0))$	$I_{mj}(f_1^{[n]}(0))$
0	1.000	0.000	1.000	0.000
1	0.000	0.000	-.653	8.600
2	-.223	7.958	-70.920	-11.293
3	0.000	0.000	118.52	-629.29
4	-176.142	-14.930	5420.0	1182.0
5	0.000	0.000	-11021.	51323.
6	295.78	-7546.5	-476006.	-97416.



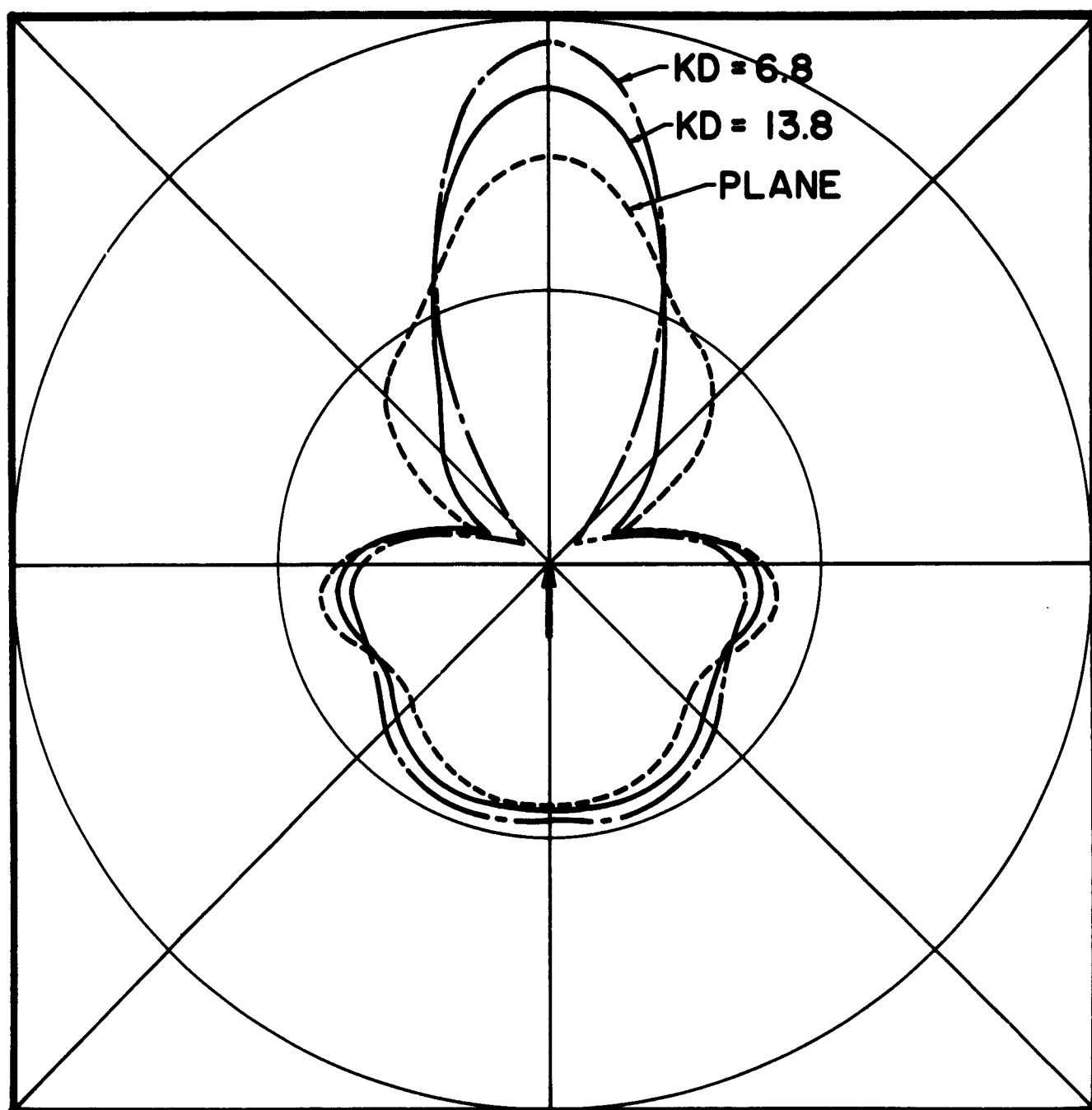
Far Field Amplitude  
Line Source at Various Distances  $D$   
Scattered from Cylinder  $ka = 1$ .  
Neumann Boundary Condition

Figure 3



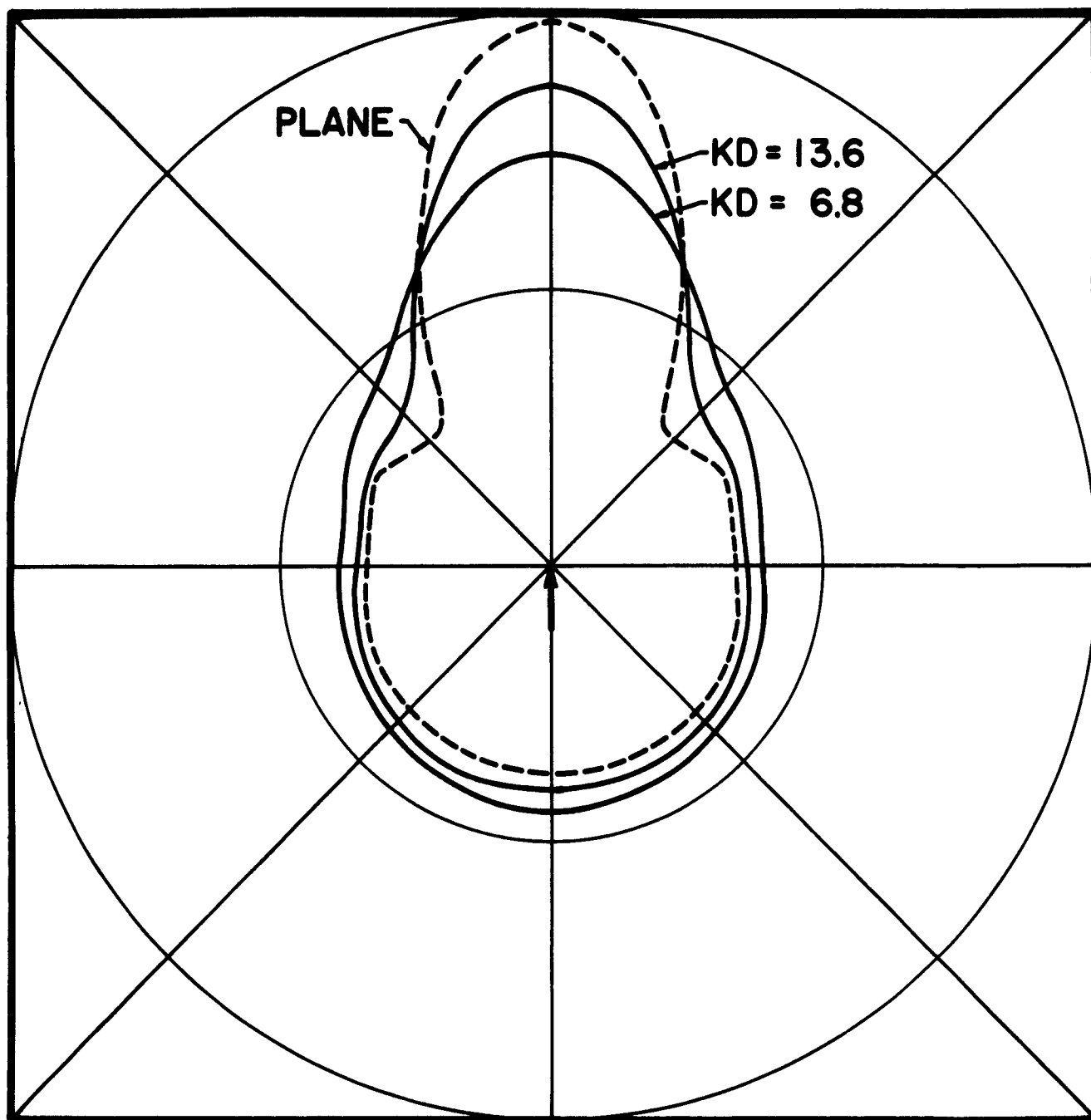
Far Field Amplitude  
Line Source at Various Distances  $D$   
Target is Cylinder  $ka = 1$ .  
Dirichlet Boundary Conditions

Figure 4



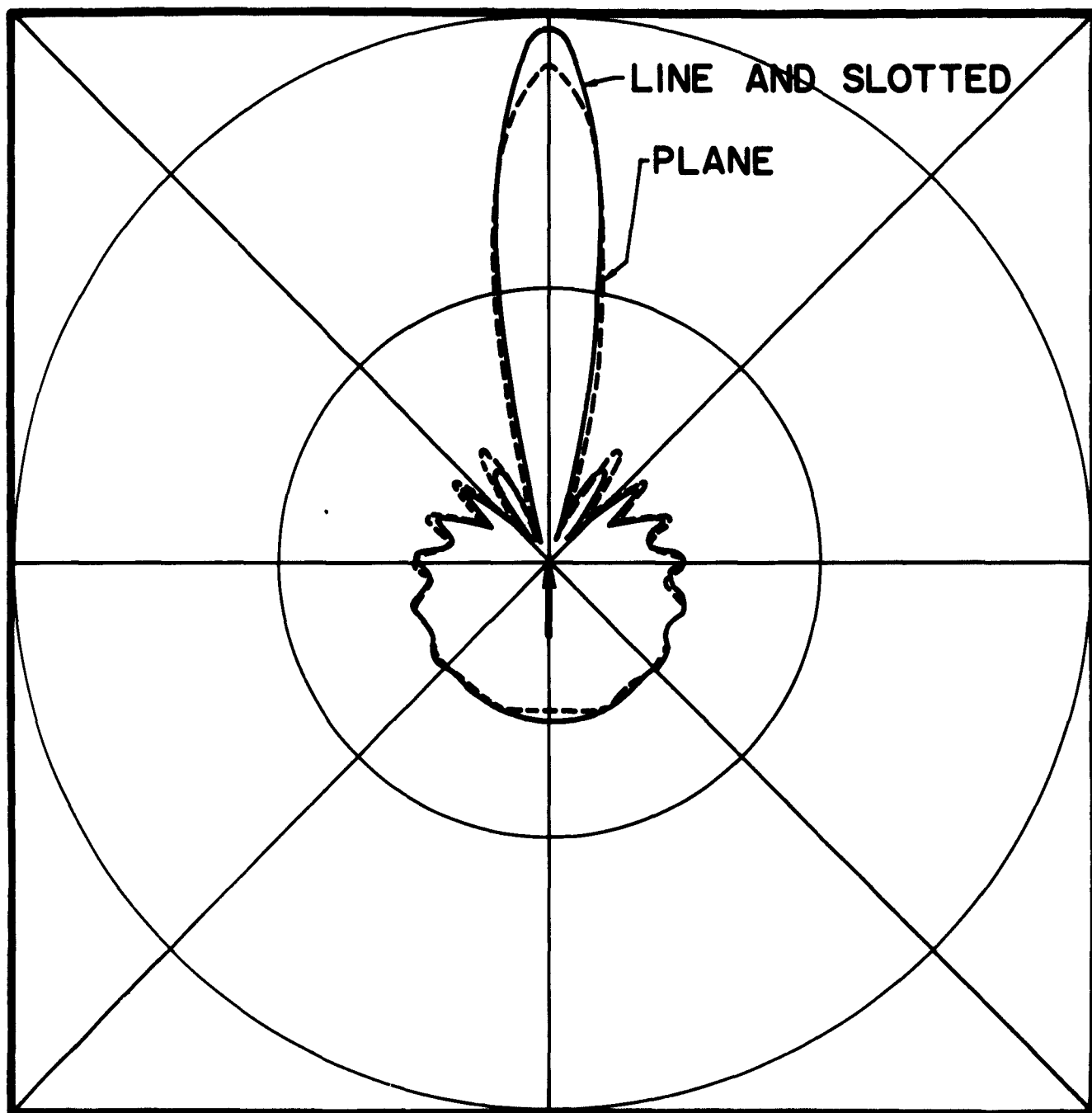
Line Source With Cylindrical Target  $ka = 3.4$   
Far Field Amplitude, Neumann Boundary Condition

Figure 5



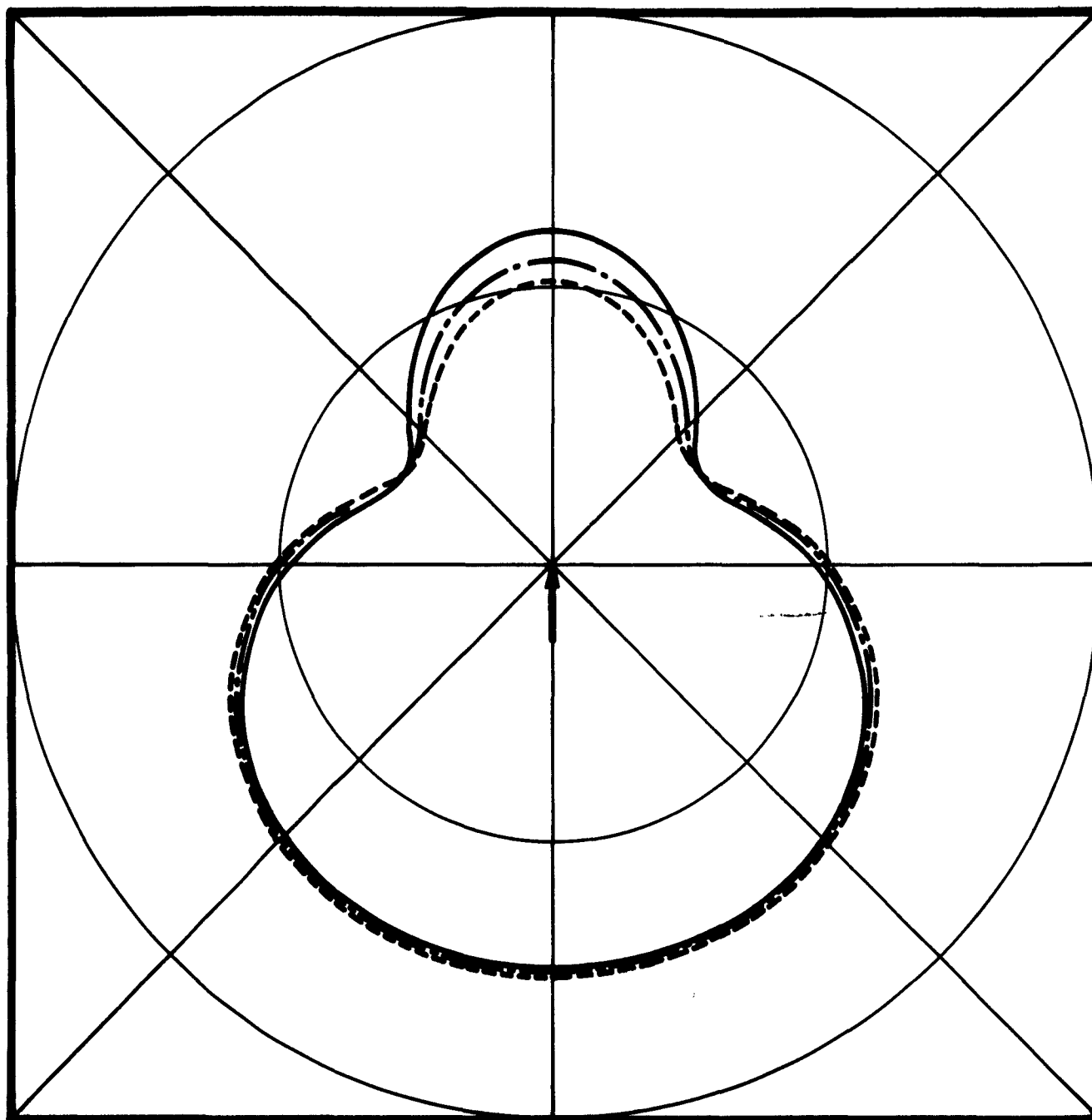
Line Source With Cylindrical Target  $ka = 3.4$   
Far Field Plotted  
Dirichlet Boundary Condition

Figure 6



Far Field Amplitude  $ka = 10$ .  $kd = 100$ .  
Neumann Conditions

Figure 7



Far Field Amplitude

Sources:

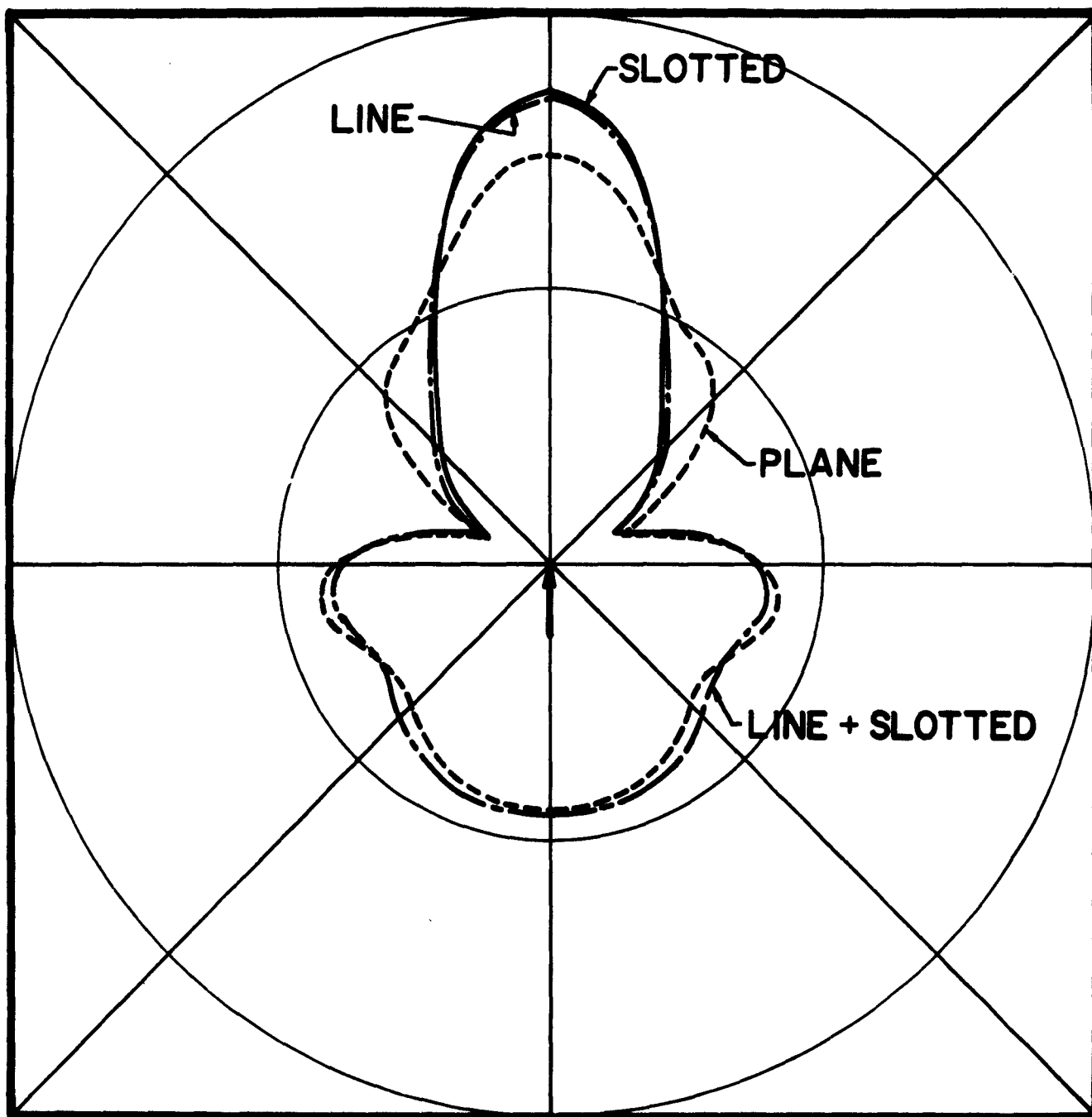
- Plane Wave
- Line Source  $kd = 10$ .
- Symmetric Slotted Cylinder  $kd = 10$ .

Target: Cylinder  $ka = 1$ .

Figure 8

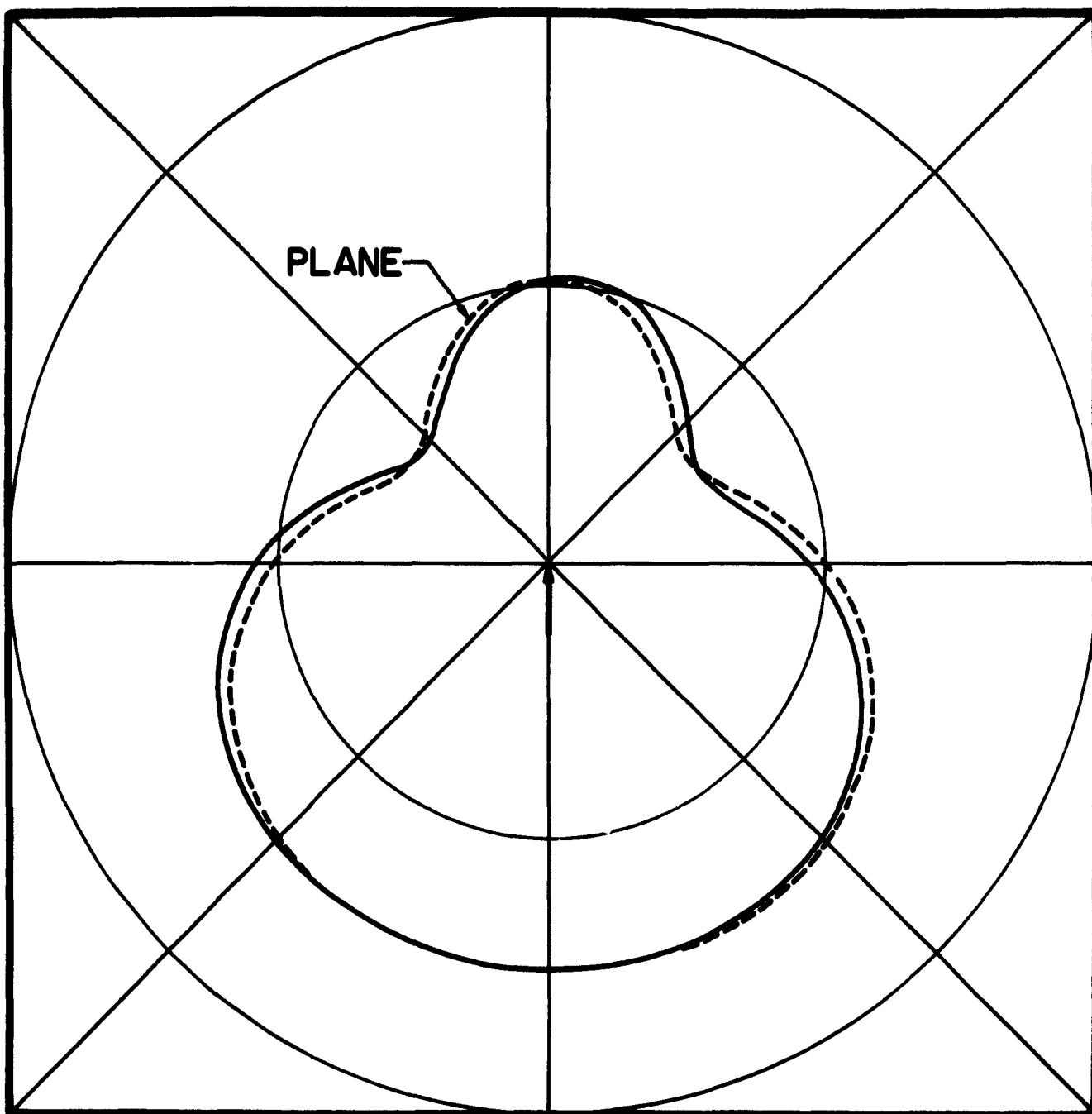
Neumann Boundary Conditions





Far Field Amplitude  $ka = 3.4$   
 Line Source  $kd = 17$   
 Slotted Source  $kc = 2. \theta_0 = .05r \quad kd = 17$

Figure 9



Far Field Amplitude

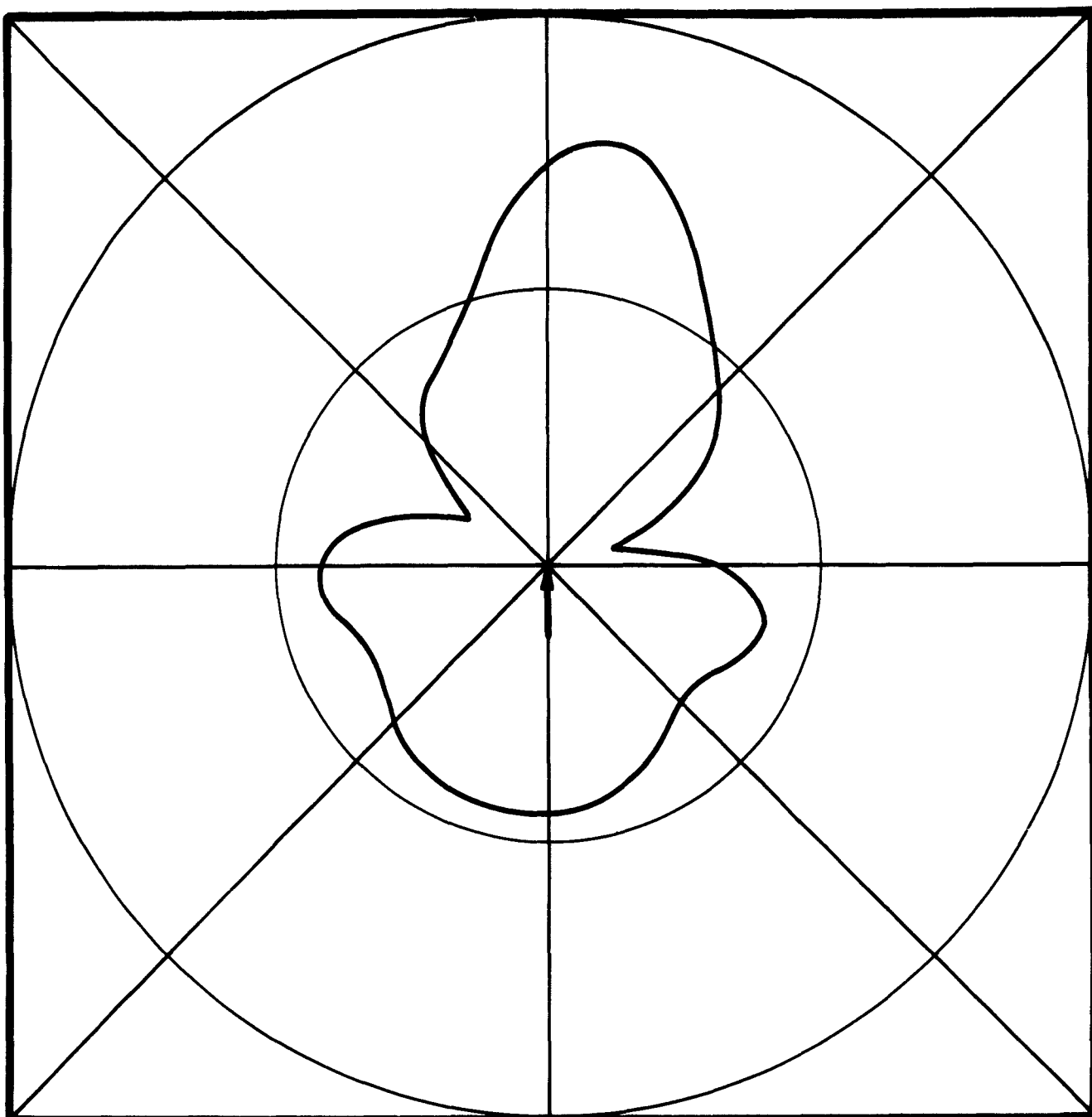
Rotated Slotted Source  $kc = 8$ .  
 Scatterer is Cylinder  $ka = 1.0$  at Distance  
 $kd = 100$ . is Far Field of Plane Wave.

Width .001r.



Neumann Boundary Condition

Figure 10




Source: Slotted, With  $kc = 8.0$ , Distance  $kd = 68.0$ , Orientation   
 Scatterer: Cylinder, With  $ka = 3.4$ , Neumann Boundary Conditions

Figure 11

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